The Sentiment Channel of Fiscal Policy

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Presented by: Rafael Lincoln
Macro Reading Group

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- Standard NK models: fiscal multiplier ≈ 0.5—investment crowded out
- Empirical estimates suggest multiplier ≈ 1—what's missing?

This Paper: Sentiment Doubles the Fiscal Multiplier

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- Novel Italian micro-data linking procurement, expectations, and balance sheets
- Causal evidence: gov spending makes firms overoptimistic (forecast errors)
- Optimism spills to unrelated domains (cross-domain extrapolation)

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Theoretical contribution: Develop tractable theory of Cross-Domain Extrapolation (XDE)

Quantitative contribution:

- Het-firm NK model with behavioral expectations: multiplier ↑ from 0.49 to 1
- Financial frictions limit the channel: multiplier 34% smaller in credit crunches

Related Literature

Fiscal multipliers

- Blanchard–Perotti (2002), Ramey (2011), Ben Zeev–Pappa (2017), Nakamura–Steinsson (2014)

Behavioral macro / diagnostic expectations

- Bordalo et al. (2018, 2025), Maxted (2023), Angeletos–Lian (2021)

Financial frictions and investment

- Fazzari et al. (1988), Ottonello–Winberry (2020)

This paper: first causal evidence on the sentiment channel of fiscal policy; alternative to HANK for generating large multipliers

Roadmap

1. **Data and Empirical Evidence**
   - Data
   - Identification
   - Overreaction
   - Cross-domain extrapolation
2. **Model**
   - Behavioral theory (XDE)
   - Partial equilibrium investment
   - General equilibrium
3. **Calibration and Main Results**
   - Calibration
   - Fiscal experiment
   - Decomposition
   - State dependence
4. **Conclusion**

Empirical Evidence

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2015 Stability Pact reform: base years for municipal surplus targets changed (2009–2011 → 2010–2012) ⇒ unanticipated fiscal space; municipal "shift":

Stability Pact (appendix)

$$g_m \equiv \log(\text{Exp}_{m,2009}) - \log(\text{Exp}_{m,2012})$$

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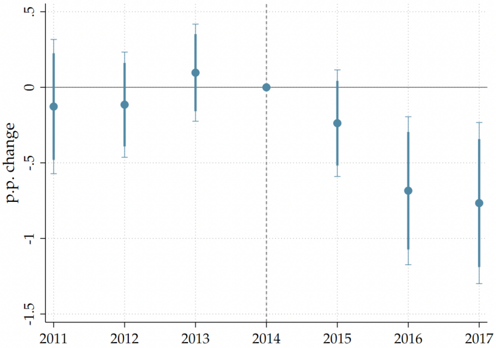
Key variables:

- Outcome: $\mathcal{E}_{it} = \log(\text{Sales}_{it}) - \log(\mathbb{F}_{t-1}\text{Sales}_{it})$; $\quad \mathcal{E} < 0 =$ overoptimism
- Shift-share: $z_i = \sum_m s_{im}\, g_m$, with $s_{im} = \frac{\sum_{t=2012}^{2014} \text{Sales}_{imt}}{\sum_{t=2012}^{2014} \sum_{m'} \text{Sales}_{im't}}$ (firm i's share of procurement from m, pre-reform)

Summary statistics   Identification (appendix)

Government Spending Makes Firms Overoptimistic



Notes: 90 and 95% CI. Y-axis: forecast error (pp). Negative = overoptimism.

- Event study (figure): sales forecast errors on $z_i \times \mathbf{1}(t)$

$$\mathcal{E}_{it} = \alpha_i + \gamma_{s(i)t} + \sum_h \beta_h (\mathbf{1}_{t=h} \times z_i) + \varepsilon_{it}$$

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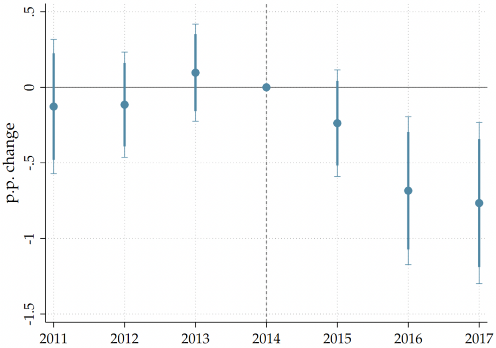

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Regression

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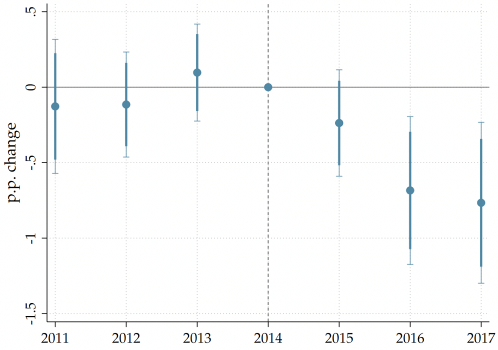

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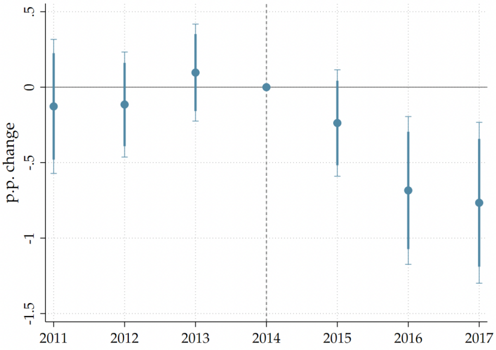

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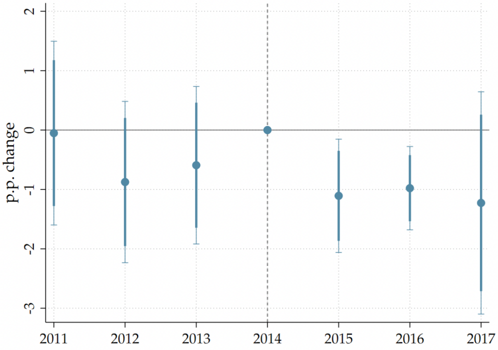
- Firms become overly optimistic—reject rational expectations Table 1

Optimism Spills Over: Cross-Domain Extrapolation



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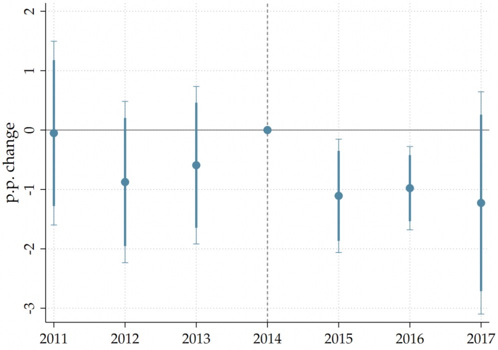

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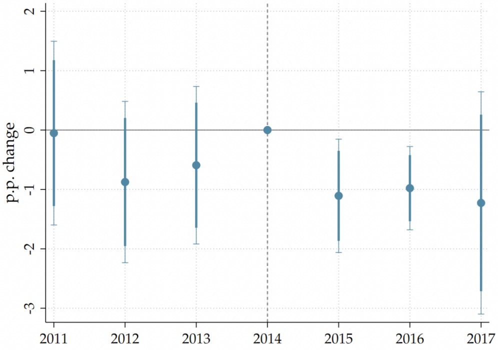

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- Export forecast errors respond negatively despite no export channel
- "Cross-domain": optimism from public demand spills to private beliefs

Empirical Takeaways and Why a Model?

Critical takeaways:

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Empirics show sentiment exists, but cannot answer counterfactual questions:

1. How large is the fiscal multiplier when firms have behavioral biases?
2. Does sentiment-induced optimism overturn crowding-out of investment?
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Need a structural model that delivers:

- Theory for overreaction and cross-domain extrapolation in sales expectations
- Investment decisions with financial constraints (sentiment $\rightarrow$ capital spending)
- Aggregate fiscal policy experiments (multipliers, state dependence)

Model

Behavioral Theory: Cross-Domain Extrapolation (XDE)

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- Cross-domain Extrapolation:
  - **Representativeness**: recent shocks are overweighted (DE)
  - **Salience**: total demand $x_t = z_t + g_t$ triggers overweighting;
- Actual vs perceived laws of motion:

$$dz_t = (-\mu_z z_t + \qquad\quad) \, dt + dZ_t,$$
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- **Sentiment**: $S_t = \int_{-\infty}^{t} e^{-\kappa(t-s)}(dZ_s + dG_s)$: exponentially weighted past shocks
- **Bias**: Captures $\text{cov}(x_t, \text{shock})$ + precision + strength of representativeness
- $\chi = 0$ nests RE; $\chi > 0 \Rightarrow$ overreaction to recent news Proposition 1

Partial Equilibrium: Firm's Investment Problem

Continuum of monopolistically competitive firms invest ι and pay dividends d:

Full problem

$$V(k_0, S_0, g_0, z_0) = \max_{\{\iota_t\}} \tilde{\mathbb{E}}_0 \left[\int_0^\infty e^{-rt} \big(d_t + \Gamma(d_t) \big) \, dt \right]$$

$$\text{s.t.} \quad \dot{k}_t = \iota_t - \delta k_t, \qquad d_t = \pi(k, g, z) - \iota_t - \Phi(\iota_t, k_t), \quad + \text{Perceived LoMs}$$

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Financial frictions:

- Adjustment costs: $\Phi(\iota, k) = \frac{\varphi}{2}\left(\frac{\iota}{k} - \delta\right)^2 k \implies$ Fwd-looking ι (Q-theory)
- Financing costs: $\Gamma(d) = -\frac{\zeta}{2} d^2 \cdot \mathbf{1}(d < 0) \implies$ External finance premium

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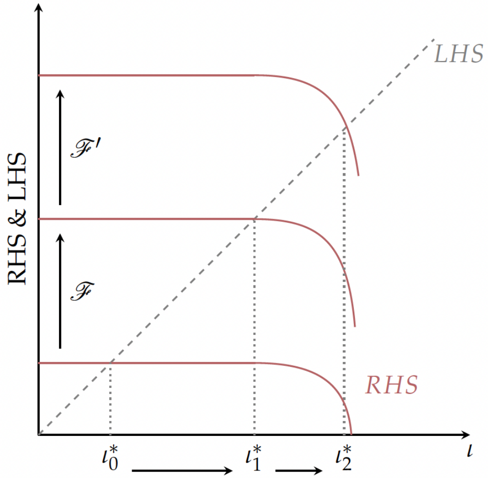
Naïve firms: S enters state space but firms don't track dS_t. HJB: Full HJB

$$rV(s) = \max_\iota \left[\pi(k, g, z) - \iota - \Phi + \Gamma(d) + (\iota - \delta k) V_k + \underbrace{\text{drift/diffusion}}_{k,g,z} + \underbrace{\chi_z S\, V_z + \chi_g S\, V_g}_{\text{rationality wedge}} \right]$$

Optimal Investment Rule (Proposition 3)



FOC $\Rightarrow$ optimal investment rule:

$$\Phi_\iota(\iota, k) = \frac{\overbrace{q_t + \mathcal{F}_t}^{\tilde{q}_t}}{1 + \Gamma'(d)} - 1$$

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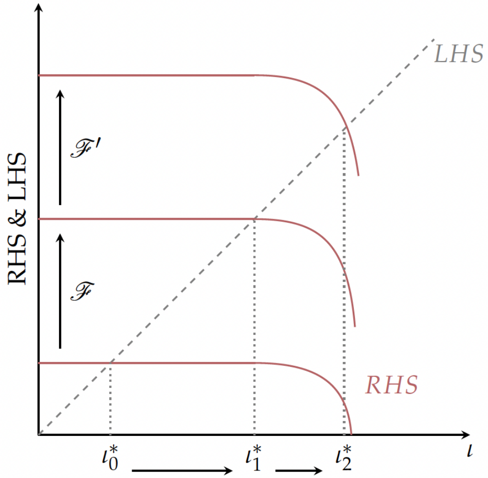

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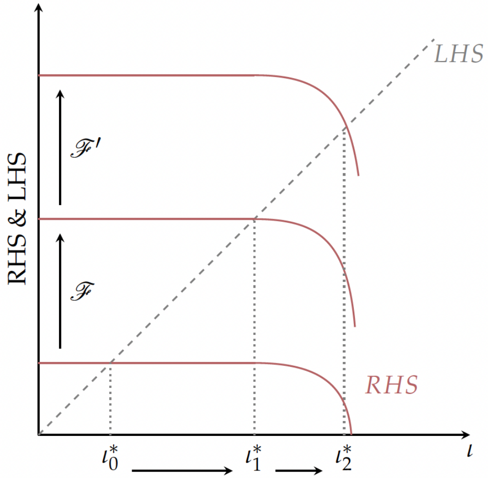

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- $\mathcal{F}_t > 0$: sentiment raises perceived $q \Rightarrow$ more investment
- $\Gamma'(d)$ large when $d < 0$: financial frictions dampen the response

$$MPIS \equiv \frac{\partial \iota}{\partial S} = \frac{\partial \iota}{\partial \mathcal{F}} \frac{\partial \mathcal{F}}{\partial S}$$

Derivation

General Equilibrium: Model Overview

Setting: Heterogeneous-firm New Keynesian model

Households
consume C, save A
supply labor L

Key: Firms face idiosyncratic demand shocks (z, g); invest with behavioral bias (XDE). Prices determined in GE. Perfect foresight over aggregates; XDE over idiosyncratic shocks.

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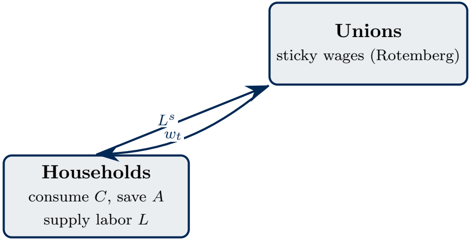

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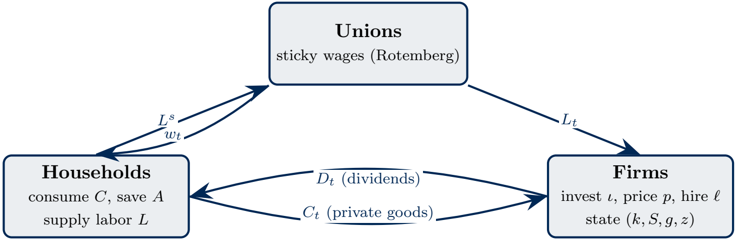

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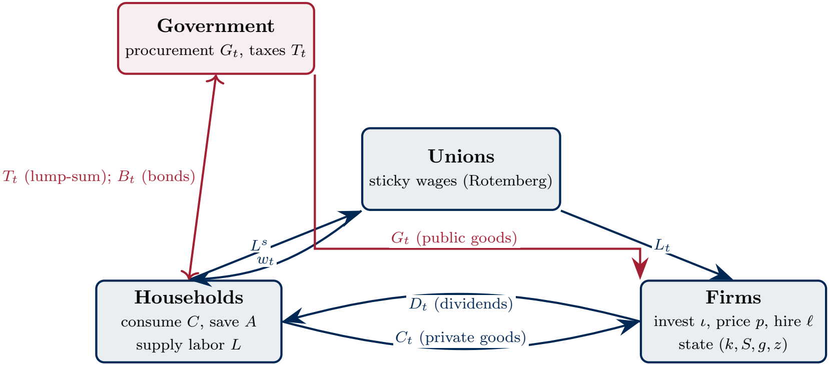

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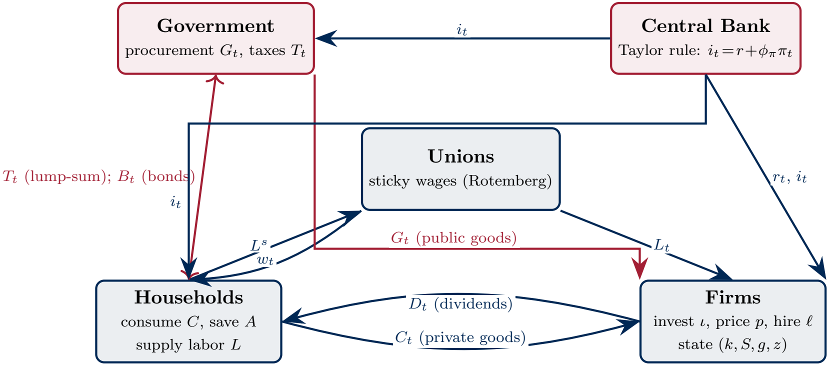

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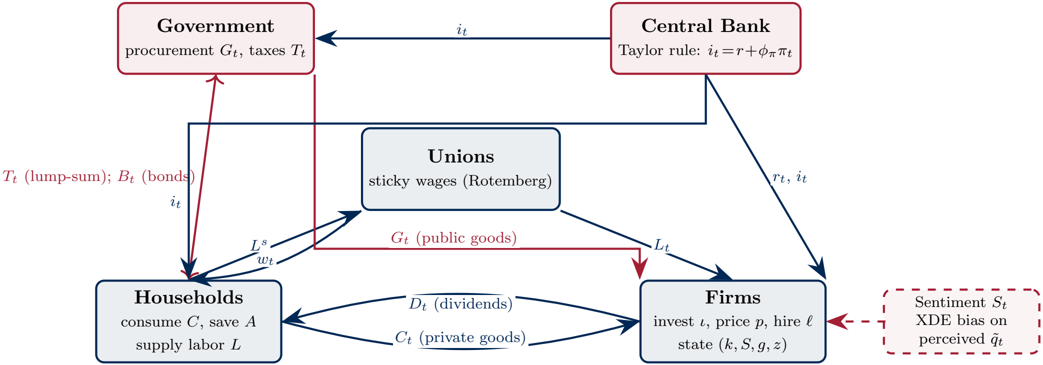

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General Equilibrium

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$$\max_{C_t, A_t, L_t} \int_0^\infty e^{-\rho t} \big(u(C_t) - v(L_t) \big) \, dt \quad \text{s.t.} \quad \dot{A}_t = w_t L_t + r_t A_t + D_t - T_t - C_t$$

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- **Unions** (sticky wages, Rotemberg): Wage NKPC

$$\max_{\{\pi^w_{kt}\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \Big(u(C_t) - v(\ell_{kt}) - \frac{\psi}{2} (\pi^w_{kt})^2 L_t\Big)\, dt,$$

$$\Longrightarrow \varepsilon_w v'(L) L - (\varepsilon_w - 1) u'(C) w L + \psi(\dot{\pi}^w - \rho\, \pi^w) = 0$$

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Firm in GE

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  Firm in GE

- **Monetary policy**: Taylor rule $i_t = r + \phi_\pi \pi_t$;
- **Fiscal policy**: balanced budget, $\dot{B}_t = r_t B_t + G_t - T_t$

General Equilibrium

- **Households**: Ricardian Behavior
$$\max_{C_t, A_t, L_t} \int_0^\infty e^{-\rho t}\big(u(C_t) - v(L_t)\big)\, dt \quad \text{s.t.} \quad \dot{A}_t = w_t L_t + r_t A_t + D_t - T_t - C_t$$
- **Unions** (sticky wages, Rotemberg): Wage NKPC
$$\max_{\{\pi^w_{kt}\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t}\Big(u(C_t) - v(\ell_{kt}) - \frac{\psi}{2}(\pi^w_{kt})^2 L_t\Big)\, dt,$$
$$\Longrightarrow \varepsilon_w v'(L)L - (\varepsilon_w - 1)u'(C)wL + \psi(\dot{\pi}^w - \rho\,\pi^w) = 0$$
- **Firms**: price setting (flex) and labor demand; $y_{it} = k_{it}^{\alpha}\ell_{it}^{1-\alpha}$ production
Firm in GE
- **Monetary policy**: Taylor rule $i_t = r + \phi_\pi \pi_t$;
- **Fiscal policy**: balanced budget, $\dot{B}_t = r_t B_t + G_t - T_t$

Mean-field game: perfect foresight over aggregates (MIT shock); time-indexed HJB; KFE uses true laws of motion Definition GE HJB KFE

Calibration: Empirically Disciplined Parameters

Param.	Description	Val.	Target / source
Externally calibrated			
ρ	Discount rate	0.02	2% steady-state r
γ	EIS	1	Standard
φ	Frisch elasticity	1	Chetty et al. (2011)
ω	HH labor disutility	1	Standard
α	Capital share	0.33	Standard
ε	Demand elasticity	10	Standard
ε_w	Union markup	21	Schmitt-Grohé–Uribe
ϕ_π	Taylor coefficient	1.5	Standard
Internally calibrated			
δ	Depreciation	0.07	Investment rate
δ_{exit}	Firm exit rate	0.065	Emp. exit rate
μ_z, σ_z	Id. private (persist., vol.)	0.65, 0.32	Revenue process
μ_g, σ_g	Id. public (persist., vol.)	0.79, 1.32	Procurement
ϕ	Adj. cost (invest.)	10	Investment-q (P&T 2017)
ψ	Wage PC	35	Slope 0.18
ζ	External finance cost	0.81	Bond premium (GZ 2012)
χ	Cross-domain extrap.	0.94	IV Table 1
κ	Sentiment memory	1.72	Forecast-error autocorr.

(annual frequency)

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Main Results

Fiscal Experiment: 1% Procurement Shock

- 1% increase in aggregate government consumption G_t on impact

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- 1% increase in aggregate government consumption G_t on impact
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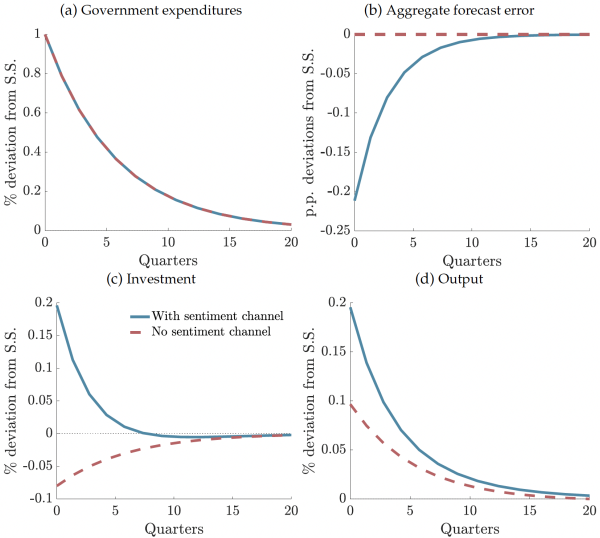
- Compare two scenarios:
  1. **No sentiment**: shut down XDE ($\chi = 0$)
  2. **With sentiment**: endogenous S_t responds to procurement

Benchmark NK: Investment Crowded Out (Multiplier ≈ 0.5)



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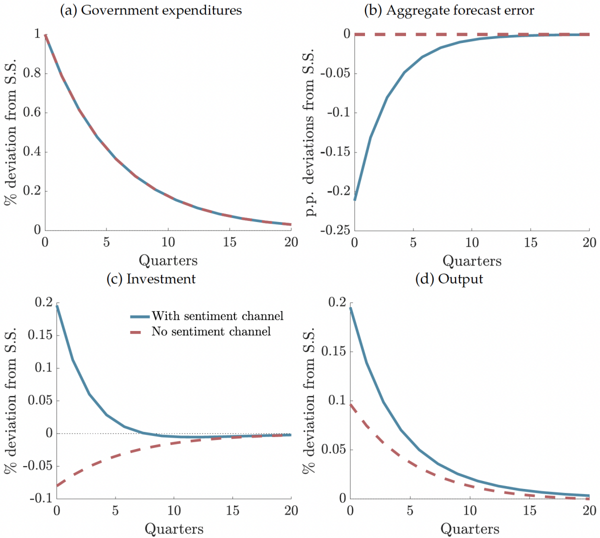

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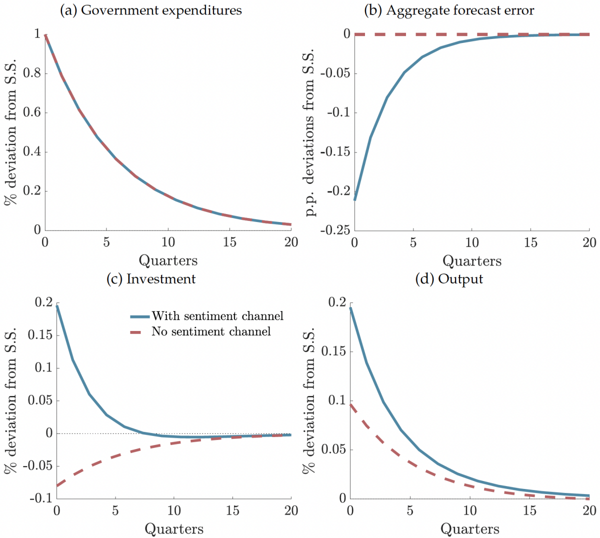

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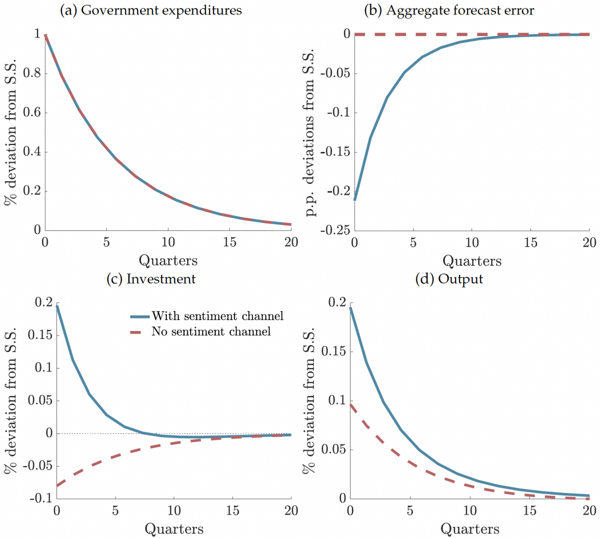

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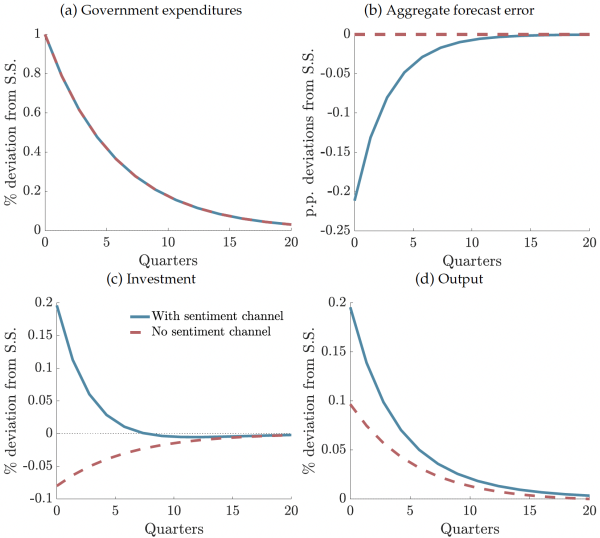
- Impact multiplier ≈ 0.49: standard NK with capital

With Sentiment: Investment Crowded In (Multiplier ≈ 1)



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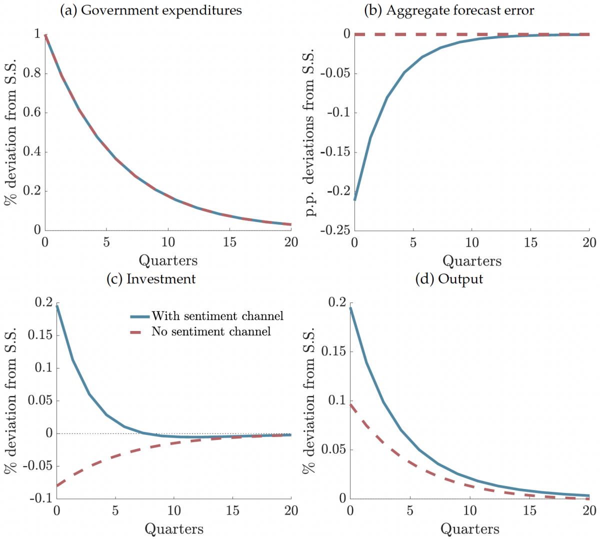

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- Investment crowded in despite higher r—sentiment dominates

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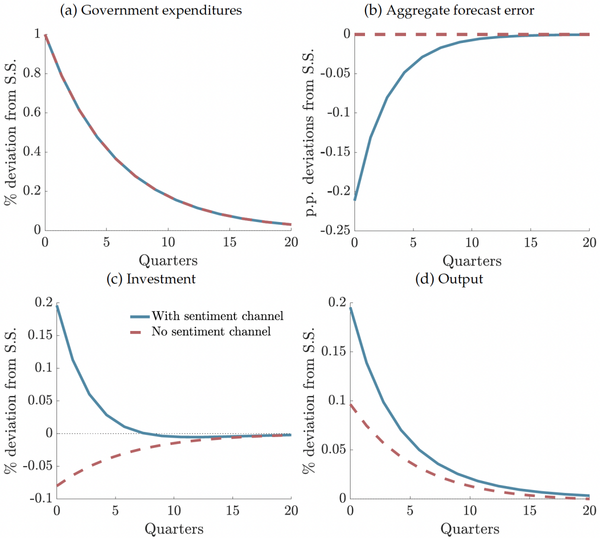

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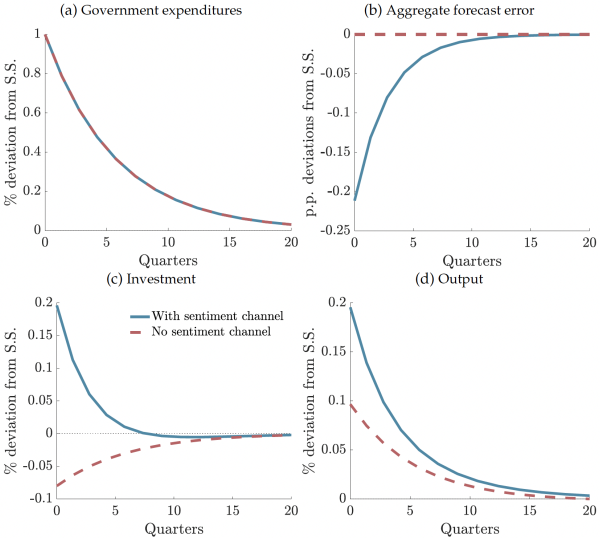

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- Output higher, consumption less crowded out, labor rises more
- **Multiplier doubles to** ≈ 1, in line with empirical estimates

Decomposing the Investment Response

Kaplan–Moll–Violante decomposition of I_t:

$$dI_0 = \underbrace{\int_0^\infty \frac{\partial I_0}{\partial G_s}\, dG_s\, ds}_{\substack{\text{direct} \\ (+)}} + \underbrace{\int_0^\infty \frac{\partial I_0}{\partial \Upsilon_s}\, d\Upsilon_s\, ds}_{\substack{\text{indirect (GE)} \\ (-\text{: } r\uparrow)}}$$

$$+ \underbrace{\int \frac{\partial \iota}{\partial S} \frac{\partial S}{\partial G_0}\, dG_0\, d\mu_0}_{\substack{\text{sentiment} \\ (+\text{ dominant})}}$$

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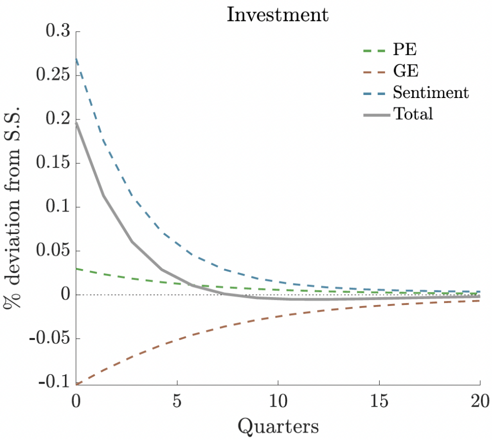
- Sentiment channel is large enough to overturn crowding-out



State Dependence: Fiscal Policy Weaker in Credit Crunches

Two recession types (normalized to same NPV output loss):

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- **Cost push**: wage Phillips curve wedge

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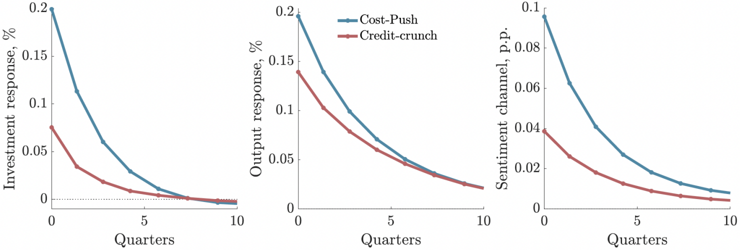

(a) Generalized IRFs, investment (b) Generalized IRFs, output (c) Generalized IRFs, sentiment channel

- GIRF $\equiv$ (recession + fiscal) − (recession only). Fiscal 34% less effective in credit crunch—MPIS is lower.

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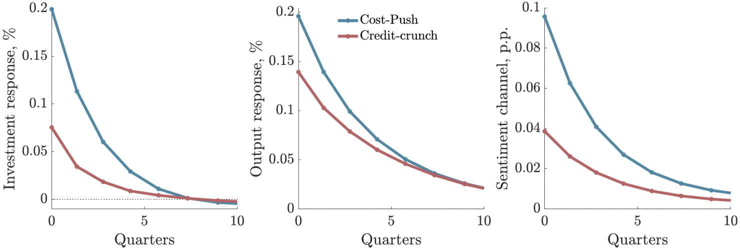

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- GIRF $\equiv$ (recession + fiscal) $-$ (recession only). Fiscal 34% less effective in credit crunch—MPIS is lower.
- Shutting down sentiment $\Rightarrow$ GIRFs equalize across recessions $\Rightarrow$ **Entire gap = sentiment channel.**

Conclusion

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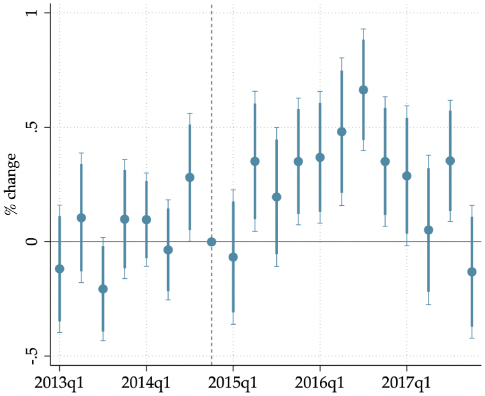
Conclusion

1. **Causal evidence**: gov spending makes firms overoptimistic (cross-domain)
2. **Quantitative model**: sentiment doubles the fiscal multiplier ($0.49 \rightarrow 1$)
3. **State-dependent**: 34% weaker in credit crunches (lower MPIS)
4. **Policy**: fiscal stimulus less potent when most needed—unless the financial system is repaired first

Appendix

Identification: 2015 Stability Pact Reform



Notes: Event study of municipal procurement on $g_m \times \mathbf{1}(t)$. 90 and 95% CI. X-axis: year relative to reform. Y-axis: procurement spending (log).

Possible Identification Issues:

- **Pre-trends**: no differential procurement before 2015

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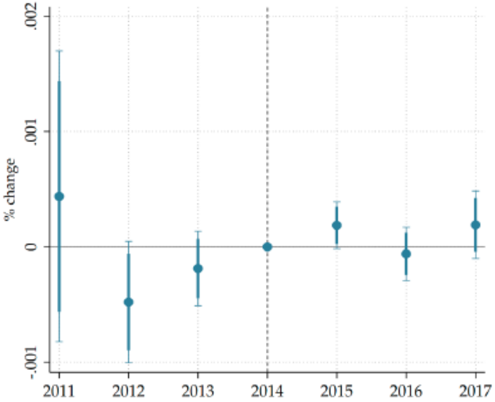

Notes: Event study of municipal tax revenues on $g_m \times \mathbf{1}(t)$. No response—reform = spending, not tax cuts. 90 and 95% CI.

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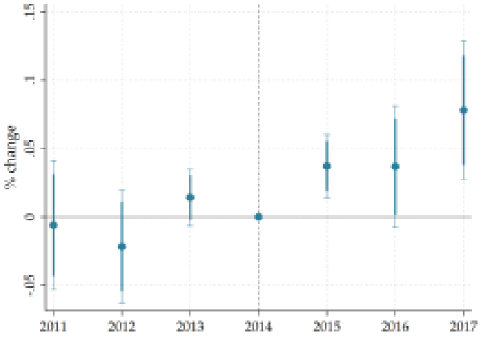

(b) Assets

Notes: Event study of firm assets on $z_i \times \mathbf{1}(t)$. No pre-trend in assets (no capacity build-up before 2015). 90 and 95% CI.

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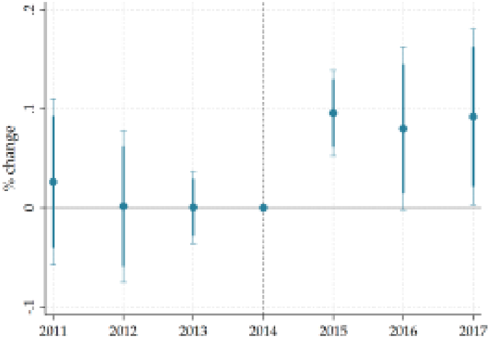

(a) Revenues

Notes: Event study of firm revenues (sales) on $z_i \times \mathbf{1}(t)$. Exposed firms see higher sales post-reform. 90 and 95% CI.

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- **Relevance**: Exposed firms see higher sales

back to Data

2015 Stability Pact Reform: Background

What is the Stability Pact?

- Italian fiscal framework: imposed budget surplus targets on municipalities (from 2007)
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Why exogenous?

- Aggregate objective (loosening) known; technical details not publicly debated
- $\Rightarrow$ plausibly unanticipated by municipalities & firms
- Large cross-sectional dispersion in $g_m = \log(\text{Exp}_{m,2009}) - \log(\text{Exp}_{m,2012})$: s.d. $= 14.7\%$

back

Definition 1 (Equilibrium)

Equilibrium back

Given a government spending shock $\{\hat{g}_i\}_{i\in[0,1]}$, an equilibrium is a sequence of

- firm policy functions $\{\iota_t(s), \ell_t(s), d_t(s)\}_{t\geq 0}$ and distributions $\{\mu_t(s)\}_{t\geq 0}$,
- prices $\{w_t, r_t, P_t^g\}_{t\geq 0}$ and aggregates $\{G_t, C_t, L_t, A_t, I_t, D_t, T_t, B_t, Y_t\}_{t\geq 0}$,

such that:

1. $\mu_0(s)$ is consistent with the endogenous response of sentiment to $\{\hat{g}_i\}_i$,
2. The path of G_t is consistent with the initial shock,
3. firms optimize given prices and sentiment (HJB),
4. Government runs balanced budget and monetary policy follows Taylor rule
5. μ_t evolves according to the KFE with initial condition μ_0,
6. Markets clear:

$$Y_t = C_t + I_t + P_t^g G_t, \qquad L_t = \int \ell_t(s)\, d\mu_t(s), \qquad A_t = B_t.$$

Firm problem in GE: private/public goods, demands, and profits

- **Two CES final-good aggregators** (private Y_t^z, public G_t):

$$Y_t^z = \left(\int_0^1 e^{z_{it}} (y_{it}^z)^{\frac{\varepsilon-1}{\varepsilon}} \, di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad G_t = \left(\int_0^1 e^{g_{it}} (y_{it}^g)^{\frac{\varepsilon-1}{\varepsilon}} \, di\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

- **Implied demands and price indices**:

$$y_{it}^z = e^{z_{it}} \left(\frac{p_{it}}{P_t^z}\right)^{-\varepsilon} Y_t^z, \quad y_{it}^g = e^{g_{it}} \left(\frac{p_{it}}{P_t^g}\right)^{-\varepsilon} G_t, \quad y_{it} = y_{it}^z + y_{it}^g;$$

$$P_t^z = \left(\int_0^1 e^{z_{it}} p_{it}^{1-\varepsilon} \, di\right)^{\frac{1}{1-\varepsilon}}, \quad P_t^g = \left(\int_0^1 e^{g_{it}} p_{it}^{1-\varepsilon} \, di\right)^{\frac{1}{1-\varepsilon}}.$$

- **Production**: $y_{it} = k_{it}^{\alpha} \ell_{it}^{1-\alpha}$, labor at w_t. Normalizing $P_t^z = 1$, the static problem delivers a **time-dependent profit function**: back

$$\pi_t(g, z, k) = \Omega_t(k) \cdot \left(e^z + \alpha_{g,t} e^g\right)^{\frac{1}{1-\alpha+\varepsilon\alpha}}, \quad \alpha_{g,t} \equiv \frac{G_t (P_t^g)^{\varepsilon}}{Y_t^z}.$$

Proposition 1: Perceived Laws of Motion under XDE

Under Cross-Domain Extrapolation with OU fundamentals:

$$d\tilde{z}_t = \left(-\mu_z z_t + \chi_z S_t\right) dt + dZ_t$$
$$d\tilde{g}_t = \left(-\mu_g g_t + \chi_g S_t\right) dt + dG_t$$

where

$$\chi_z = \chi \, \frac{\beta_z \, \sigma_z^2}{\sigma^2_{z|x}}, \qquad \chi_g = \chi \, \frac{\beta_g \, \sigma_g^2}{\sigma^2_{g|x}}$$

- β_z, β_g: OLS coefficients of z, g on salient variable $x = z + g$
- $\sigma^2_{z|x}$, $\sigma^2_{g|x}$: conditional variances
- χ: degree of representativeness ($\chi = 0 \Rightarrow$ RE)

back

Firm's Value Function (Full Problem)

$$
\begin{aligned}
V(k_0, S_0, g_0, z_0) &= \max_{\{\iota_t\}_{t\geq 0}} \tilde{\mathbb{E}}_0 \int_0^\infty e^{-rt}\big(d_t + \Gamma(d_t)\big)\, dt \\
\text{s.t.} \quad \dot{k}_t &= \iota_t - \delta k_t, \\
d_t &= \pi(k_t, g_t, z_t) - \iota_t - \Phi(\iota_t, k_t), \\
d\tilde{z}_t &= (-\mu_z z_t + \chi_z S_t)\, dt + dZ_t, \\
d\tilde{g}_t &= (-\mu_g g_t + \chi_g S_t)\, dt + dG_t, \\
dS_t &= -\kappa S_t\, dt + dZ_t + dG_t.
\end{aligned}
$$

Naïve firms: the law of motion for S (in red) is *not* imposed. Firms do not internalize or track the dynamics of sentiment. back

Full HJB Equation (PE, with Rationality Wedge)

$$
\begin{aligned}
rV(s) = \max_{\iota} \Big[& \pi(k,g,z) - \iota - \Phi(\iota,k) + \Gamma(d) \\
& + (\iota - \delta k)\,\partial_k V - \mu_z z\,\partial_z V + \tfrac{\sigma_z^2}{2}\partial_{zz}^2 V - \mu_g g\,\partial_g V + \tfrac{\sigma_g^2}{2}\partial_{gg}^2 V \\
& - \kappa S\,\partial_S V + \tfrac{\sigma_z^2+\sigma_g^2}{2}\partial_{SS}^2 V + \sigma_z^2\partial_{Sz}^2 V + \sigma_g^2\partial_{Sg}^2 V \\
& + \chi_z S\,\partial_z V + \chi_g S\,\partial_g V \Big]
\end{aligned}
$$

- Line 3 (red): contribution from dS_t—**omitted** under naïve firms
- Line 4: rationality wedge from XDE-biased perceived drifts

back

Derivation: From FOC to Proposition 3

Envelope condition for state k in the HJB:

$$(r+\delta)\tilde{q}_t = \underbrace{(\pi_k - \Phi_k)(1+\Gamma'(d))}_{\text{inst. cash yield}} + \underbrace{\mathbb{E}^{\chi}(d\tilde{q}_t/dt)}_{\text{expected capital gains}}$$

Expectation wedge:

$$\mathbb{E}^{\chi}(d\tilde{q}_t/dt) - \mathbb{E}(d\tilde{q}_t/dt) = (\chi_z \partial_z \tilde{q}_t + \chi_g \partial_g \tilde{q}_t)\, S_t$$

Feynman–Kac $\Rightarrow$ behavioral wedge in perceived marginal q:

$$\mathcal{F}_t = \int_0^{\infty} e^{-(r+\delta)s}\,(\mathbb{E}_t^{\chi} - \mathbb{E}_t)\big[(1+\Gamma')\,(\pi_k - \Phi_k)\big]\, ds$$

FOC: $\Phi_\iota(\iota, k) = \dfrac{q_t + \mathcal{F}_t}{1+\Gamma'(d)} - 1$ back

GE HJB Equation vs. Partial Equilibrium

$$
\begin{aligned}
r_t V_t(s) - \partial_t V_t(s) = \max_{\iota} \Big[&\pi_t(k, g, z) - \iota - \Phi(\iota, k) + \Gamma(d_t) \\
&+ (\iota - \delta k)\partial_k V_t - \mu_z z \, \partial_z V_t + \tfrac{\sigma_z^2}{2} \partial_{zz}^2 V_t \\
&- \mu_g g \, \partial_g V_t + \tfrac{\sigma_g^2}{2} \partial_{gg}^2 V_t \\
&+ \chi_z S \, \partial_z V_t + \chi_g S \, \partial_g V_t \Big]
\end{aligned}
$$

Differences from PE:

- $-\partial_t V_t$: time derivative (prices change over transition)
- r_t, $\pi_t(\cdot)$: time-varying aggregates
- Red line omitted (naïve firms, same as PE)

back

Kolmogorov Forward Equation (True Distribution)

$$\begin{aligned}\partial_t \mu_t(s) = &-\partial_k[(\iota_t(s) - \delta k)\mu_t] - \mu_z \partial_z[z\,\mu_t] - \mu_g \partial_g[g\,\mu_t] - \kappa \partial_S[S\,\mu_t] \\ &+ \tfrac{\sigma_z^2}{2}\partial_{zz}^2 \mu_t + \tfrac{\sigma_g^2}{2}\partial_{gg}^2 \mu_t + \tfrac{\sigma_z^2+\sigma_g^2}{2}\partial_{SS}^2 \mu_t \\ &+ \sigma_z^2 \partial_{zS}^2 \mu_t + \sigma_g^2 \partial_{gS}^2 \mu_t\end{aligned}$$

Key: KFE uses *true* laws of motion—no rationality wedge ($\chi_z S$, $\chi_g S$).

Misperceptions enter only *indirectly* through biased policy functions $\iota_t(s)$.

Biased HJB, correct KFE: hallmark of mean-field game with behavioral agents.

back

AR(1) to Ornstein–Uhlenbeck Mapping

Estimate AR(1) with firm fixed effects:

$$\log Sales^z_{it} = \alpha^z_i + \rho_z \log Sales^z_{it-1} + u^z_{it}$$
$$\log Sales^g_{it} = \alpha^g_i + \rho_g \log Sales^g_{it-1} + u^g_{it}$$

Map to continuous-time OU parameters:

$$\mu_x = -\log(\hat{\rho}_x), \qquad \sigma_x = \sqrt{\frac{\hat{\sigma}^2_{u,x}}{2\mu_x(1-\hat{\rho}^2_x)}}$$

for $x \in \{z, g\}$. back

Psychology Parameter Identification: (χ, κ)

Solve $F(\chi, \kappa) = (\hat{\beta}, \hat{\rho})$ where F maps candidates to simulated moments:

1. Fix (χ, κ); simulate (z_t, g_t, S_t) on monthly grid $(\Delta = 1/12)$
2. Construct perceived fundamentals at horizon τ:
   $\tilde{z}_\tau = z_0 e^{-\mu_z \tau} + \frac{\chi_z}{\mu_z} S_0 (1 - e^{-\mu_z \tau})$
3. Define forecast-error series from realized vs. perceived sales
4. Estimate by OLS: slope $\tilde{\beta}$ and autocorrelation $\tilde{\rho}$
5. Solve $F(\chi, \kappa) = (\hat{\beta}, \hat{\rho})$ via Newton–Raphson

Targets: $\hat{\beta}$ from IV (Table 1); $\hat{\rho} \simeq 0.17$ from forecast-error data back

Testing Q-Theory: Investment–Sentiment–Frictions

$$\log\left(\frac{I_{it}}{K_{it}}\right) = \gamma\,\Delta\log \text{Sales}_{it+1} + \delta\,\hat{F}_{it+1} + \beta\,\hat{F}_{it+1} \times D^{\text{score}}_{it} + \psi\,D^{\text{score}}_{it} + \Xi X'_{it} + u_{it}$$

- $\hat{F}_{it+1}$: forecast error in sales growth (sentiment proxy)
- $D^{\text{score}}_{it} = 1$ if firm in bottom half of credit score (high frictions)
- $\beta > 0$: sentiment effect on investment smaller for constrained firms

	Dependent variable: $\log \frac{Inv_{it}}{K_{it}}$				
	(1)	(2)	(3)	(4)	(5)
$\Delta \log Sales_{it+1}$	1.207***	1.415***	0.690***	1.099***	1.189***
	(0.339)	(0.171)	(0.115)	(0.136)	(0.152)
$\widehat{\mathscr{F}}_{it+1}$	-1.119***	-1.130***	-0.712***	-0.847***	-0.850***
	(0.168)	(0.136)	(0.103)	(0.115)	(0.148)
Low score$_{it}$	-0.151***	-0.163***	-0.074***	-0.194***	-0.162***
	(0.052)	(0.049)	(0.024)	(0.044)	(0.045)
$\widehat{\mathscr{F}}_{it+1} \times$ Low score$_{it}$	0.386***	0.379***	0.413***	0.283**	0.199*
	(0.135)	(0.131)	(0.104)	(0.123)	(0.106)
$\log \frac{Inv_{it-1}}{K_{it-1}}$			0.690***		
			(0.017)		

IV Specification: Overreaction (Paper Eq. 10)

$$\mathcal{E}_{it} = \alpha_i + \delta_t + \beta \log \mathrm{Sales}_{it-1} + u_{it}$$

Instrument $\log \mathrm{Sales}_{it-1}$ with $z_i \times \mathrm{Post}_t$ (excluding 2015).

	First Stage	Reduced Form	IV
$\mathrm{Post}_t \times z_i$	0.718***	-0.576***	
	(0.203)	(0.169)	
$\log Sales_{it-1}$			-0.863***
			(0.315)
Kleibergen-Paap F-stat.			11.408
Observations	3,580	3,576	3,576
Year FEs	✓	✓	✓
Firm FEs	✓	✓	✓

Notes: KP F-stat > 10. $\beta < 0$: higher instrumented sales $\Rightarrow$ more negative forecast errors (overoptimism). back

Static DiD: Cross-Domain Extrapolation (Paper Eq. 11; Table 2)

Table 2 (paper): response of export forecast errors

	Baseline (2011–2017)	Narrow (2013–2016)
$\text{Post}_t \times z_i$	-1.003^{**}	-1.379^{**}
	(0.384)	(0.352)
Observations	493	281
Firm FEs	✓	✓
Sector×year FEs	✓	✓

back

Specification (Eq. 11)

$$\mathcal{E}^X_{it} = \alpha_i + \gamma_{s(i)t} + \beta\, z_i \times \text{Post}_t + \varepsilon_{it}$$

- $\mathcal{E}^X_{it}$: export sales forecast error (exports orthogonal to procurement).
- α_i: firm FE; $\gamma_{s(i)t}$: sector×year FE.
- Why do this (besides event study)?
  - One-number average post effect ($\hat{\beta}$).
  - Robustness: narrow window around reform (2013–2016).

Summary Statistics (Table A.1)

	Obs.	Mean	p25	p50	p75
Panel A: ANAC–CADS sample					
Employment	173,261	71	4	10	24
Revenues (EUR k)	175,541	17,231	434	1,150	3,202
Assets (EUR k)	175,541	26,415	519	1,352	3,846
Leverage	175,149	12.57	2.57	5.08	11.33
Age	175,541	19	9	16	26
Panel B: ANAC–CADS–INVIND sample					
Employment	5,344	134	16	27	49
Revenues (EUR k)	5,345	33,046	2,386	4,637	10,390
Assets (EUR k)	5,345	51,300	3,282	7,372	16,478
Leverage	5,345	10	3	5	8
Age	5,345	31	16	29	37
Sales forecast error	3,761	−2.47	−19.68	−1.60	15.58

Notes: Revenues and assets in thousands of euros; employment in units. Panel B is the matched sample used for forecast-error analysis. back