Dynamic Identification in VARs

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Macro Reading Group

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Motivation

- Macroeconomists want to understand how economy responds to structural shocks
- Two main approaches, each with limitations:
  (a) **DSGE Models**: Many restrictions → prone to misspecification
  (b) **SVARs**: Fewer restrictions → controversial identification assumptions
- Two Critical Questions:
  1. Can we achieve identification using restrictions that are already standard in macroeconomic theory?
  2. Given this new identification approach, can we test the validity of traditional SVAR restrictions?

This Paper

This Paper's Innovation:

- Middle ground: Use DSGE's dynamic structure WITHOUT cross-equation restrictions
- Key insight: If shocks follow independent AR(1) processes (standard in DSGE), this ”dynamic fingerprint” is sufficient for identification

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Preview of Results:

- Identification of structural shocks without the usual restrictions/instruments
- Method to test common SVAR restrictions
- Applications show some classic identifications hold, others don't

Related Literature

Builds on Foundational Work

- Komunjer & Ng (2011): Identification in linearized DSGE models
- McGrattan (2010): State-space identification in specific RBC models
- Pagan & Robinson (2019): Statistical restrictions in DSGE vs SVAR

Key Contributions Beyond Existing Work

- Broader class of models than McGrattan (2010)
- Opposite approach to Bai & Wang (2015): loadings $\times$ dynamics
- Extends Gourieroux & Jasiak (2022)

Related SVAR literature: Sims (1980), CEE (1999), Blanchard-Perotti (2003), Blanchard-Quah (1989), Shapiro-Watson (1988), Uhlig (2005), Stock-Watson (2012), Gertler-Karadi (2015)

Roadmap

1. **Introduction**
2. **Establishing Dynamic Identification**
   - Identification Challenge
   - Example: NK model and Identification with D-SVAR
   - Proving local identification
3. **Evaluating DSGE and SVAR Models**
   - Estimation and Inference
   - Example: CEE (1999)
4. **Conclusion**

Establishing Dynamic Identification

Mapping: DSGE $\rightarrow$ State-Space

- DGP $\mathbb{D}(\cdot \mid \theta)$ is a Linear Rational Expectations Model:

$$\begin{aligned} X_t &= M_1(\theta) X_{t-1} + M_2(\theta) \mathbb{E}_t[X_{t+1}] + M_3(\theta) Z_t \\ Z_t &= R(\theta) Z_{t-1} + \varepsilon_t; \quad \varepsilon_t \sim \mathcal{N}(0, \mathbf{I}_{n_z}) \end{aligned} \tag{1}$$

$X_t = \begin{bmatrix} Y_t & K_t \end{bmatrix}'$ Observables / Endogenous; $Z_t =$ Exogenous

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$X_t = \begin{bmatrix} Y_t & K_t \end{bmatrix}'$ Observables / Endogenous; $Z_t =$ Exogenous

- Under Saddle Path, State-Space representation $\mathcal{S}$ follows:

$$\begin{aligned} K_{t+1} &= GK_t + FZ_t; \\ Y_t &= \Pi_{yk}K_t + \Pi_{yz}Z_t; \\ Z_t &= RZ_{t-1} + \varepsilon_t; \end{aligned} \tag{2}$$

- Establish a mapping $\psi = \mathcal{G}(\theta)$, $\psi = (\mathrm{Vec}(G), \mathrm{Vec}(F), \mathrm{Vec}(\Pi_{yk}), \mathrm{Vec}(\Pi_{yz}), \mathrm{Vec}(R))$

Mapping: State-Space $\rightarrow$ Reduced Form

- *Simplification*: Assume $X_t = Y_t$ (all variables are observed) General Form

$$\begin{aligned} Y_t &= GY_{t-1} + FZ_t \\ Z_t &= RZ_{t-1} + \varepsilon_t \end{aligned} \tag{3}$$

- Under invertibility, recover VAR(2) form:

$$Y_t = \underbrace{(G + FRF)^{-1}}_{\equiv\Gamma_1} Y_{t-1} + \underbrace{(-FRF^{-1})}_{\Gamma_2} Y_{t-2} + \underbrace{F\varepsilon_t}_{a_t} \tag{4}$$

- Estimate Reduced-form VAR(2), mapping $(\Gamma_1, \Gamma_2, \Sigma_a) = \Psi(\psi)$ where:

$$\Sigma_a = FF'$$

(Local) Identification Challenge

- **Issue**: Ψ has $3n^2$ unknowns $\times$ $3n^2 - \frac{n(n-1)}{2}$ independent equations
- *Usual Approach*: restrictions on the loading matrix F:
  - Short-run Restrictions (Sims, Christiano-Eichenbaum-Evans, Blanchard-Perotti, . . .)
  - Long-run Restrictions (Shapiro-Watson, Blanchard-Quah, . . .)
  - Sign Restrictions (Uhlig, . . .)
- This paper: Identification through dynamic structure of shocks, R (D-SVAR)
  - *Intuition*: Structure of shocks Z_t leaves "Dynamic Fingerprint" on IRFs/Covariances useful for identification

Example: Identification of Supply & Demand shocks

- Consider the Following 3-eq. NK model ($Y_t = \begin{bmatrix} y_t & \pi_t \end{bmatrix}'$ and $Z_t = \begin{bmatrix} z_{1,t} & z_{2,t} \end{bmatrix}'$):

$$\begin{cases} y_t = \mathbb{E}_t y_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1}) + z_{1,t}, \\ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + z_{2,t}, \\ i_t = \phi_\pi \pi_t, \end{cases} \implies \begin{cases} Y_t = \Pi_{yz} Z_t, \\ Z_t = R Z_{t-1} + \varepsilon_t. \end{cases} \tag{5}$$

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- Model admits SVMA(∞) form

$$Y_t = \sum_{j=0}^{\infty} \Theta_j \varepsilon_{t-j}, \quad \Theta_j = \Psi_j(R, \Pi_{yz}) \tag{6}$$

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- **Goal**: Recover $\psi = (r_{11}, r_{12}, r_{21}, r_{22}; \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$ from ACGF . Cases:
  1. R is a Diagonal matrix
  2. R is a Lower (identically upper) triangular matrix
  3. R is a Symmetric Matrix
  4. R is a Block Diagonal Matrix with blocks corresponding to cases 1 and 2

Dynamic Identification: Cases 1 & 2

- **Case 1**: $r_{12} = r_{21} = 0$. Recover ψ through ACGF iff $r_{11} \neq r_{22}$,

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \sum_{i=0}^{\infty} \begin{bmatrix} r_{11}^i & 0 \\ 0 & r_{22}^i \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-i} \\ \varepsilon_{2,t-i} \end{bmatrix}.$$

Intuition: As long as $r_{11} \neq r_{22}$, y_t, π_t have different dynamic properties

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- **Case 2**: $r_{12} = 0$ (or $r_{21} = 0$). Recover ψ through ACGF iff $r_{21} \neq 0$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \sum_{i=0}^{\infty} \begin{bmatrix} r_{11}^i & 0 \\ r_{21} S(i) & r_{22}^i \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-i} \\ \varepsilon_{2,t-i} \end{bmatrix}, \qquad S(i) = \begin{cases} \frac{r_{11}^i - r_{22}^i}{r_{11} - r_{22}}, & r_{11} \neq r_{22} \\ i r_{11}^{i-1}, & r_{11} = r_{22} \end{cases}$$

Intuition: Distinguish dynamics of π_t, y_t through effect r_{21}

Dynamic Identification: Cases 3 & 4

- **Case 3**: Consider $r_{12} = r_{21} \neq 0$. Recover ψ by ACGF iif $r \neq 0, r_{11} = r_{22} = \rho$

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \sum_{i=0}^{\infty} \begin{bmatrix} \lambda_+^i + \lambda_-^i & \lambda_+^i - \lambda_-^i \\ \lambda_+^i - \lambda_-^i & \lambda_+^i + \lambda_-^i \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-i} \\ \varepsilon_{2,t-i} \end{bmatrix}, \quad \lambda_\pm = \rho \pm r$$

Intuition: Different decay rates $\lambda_\pm$ will generate distinct dynamic properties

Dynamic Identification: Cases 3 & 4

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- **Case 4**: Add Monetary Shock $z_{3,t}$ to the Taylor Rule, assume $R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix}$

$$y_t = \pi_{11}\varepsilon_{1,t} + \pi_{12}\varepsilon_{2,t} + \pi_{13} \sum_{i=0}^{\infty} \rho^i \varepsilon_{3,t-i} \underbrace{\Longrightarrow}_{ACGF} \begin{cases} \gamma_y(0) & = \pi_{11}^2 + \pi_{12}^2 + \frac{\pi_{13}^2}{1-\rho^2} \\ \gamma_y(0) & = \rho^h \frac{\pi_{13}^2}{1-\rho^2}, \quad h > 0 \end{cases}$$

Partial Identification: Recover $\{\pi_{13}, \pi_{23}, \pi_{33}\}$ from ACGF, but effects of $\varepsilon_{1,t}; \varepsilon_{2,t}$ cannot be identified

In a Nutshell: *Why does it work?*

- Recast $\mathcal{S}(\psi)$ (2) as ABCD form, with state $S_t = \begin{bmatrix} K_t & Z_t \end{bmatrix}'$

$$\begin{aligned} S_{t+1} &= AS_t + B\varepsilon_{t+1} \\ Y_{t+1} &= CS_t + D\varepsilon_{t+1} \end{aligned} \tag{7}$$

$\psi = (\mathrm{Vec}(A)', \mathrm{Vec}(B)', \mathrm{Vec}(C)', \mathrm{Vec}(D)', \mathrm{Vech}(\Sigma_\varepsilon)') \in \Psi$, with
$A = \begin{bmatrix} G & F \\ 0 & R \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I_{n_z} \end{bmatrix}$, $\Pi = \begin{bmatrix} \Pi_{yk} & \Pi_{yz} \end{bmatrix}$, $C = \Pi A$, $D = \Pi B$.

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- **Local Identification**: $\mathcal{S}(\psi)$ is uniquely determined by ACGF for multivariate covariance-stationary $\{Y_t\} \mid \psi$, *i.e.*, no observational equivalence (OE). Formal Definition

- To rule out OE in $\mathcal{S}(\psi)$, prevent rotation ambiguity (OE via state/shock relabeling)

In a Nutshell: *Why does it work?*

- A class of OE $\mathcal{S}$ processes is defined by T, U full-rank matrices such that

$$\tilde{A} = TAT^{-1}, \quad \tilde{B} = TBU, \quad \tilde{C} = CT^{-1}, \quad \tilde{D} = DU, \quad \tilde{\Sigma}_\varepsilon = U^{-1}\Sigma_\varepsilon U^{-1\prime}$$

  - T is similarity transform, $\widetilde{S}_t = T^{-1}S_t$ (i.e. rotation of the state)
  - U reparametrizes shocks, $\widetilde{Z}_t = UZ_t$ (keeping scale, $\Sigma_\varepsilon = \Sigma_{\widetilde{\varepsilon}}$)

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- **Goal**: Ensure local uniqueness in this class (identify ψ)
  1. Under *Stability & Separation* + *non-redundancy* + assumption $\Sigma_\varepsilon = I$,
     - Recover $\{G, \Pi_{yk}\}$ (Prop. 1)
     - Reduce OE to an **orthonormal** U (Prop. 2)

     Issue: F still not identified since $\tilde{Z}_t = UZ_t$ (right-rotation) is OE to Z_t.
  2. Place restrictions on R to ensure $U = I \implies$ pin down $\{F, \Pi_{yz}\}$

Evaluating DSGE and VAR Models

Estimation of D-SVAR

- **Mapping Reduced-form to SS**: Recall SS (3) with mapping $\psi = \Psi(\eta)$, where $\eta = (\mathrm{Vec}(\Gamma_1), \mathrm{Vec}(\Gamma_2), \mathrm{Vec}(\Sigma_a))$. Estimate via ML or (2-step) ALS:

$$\widehat{\psi}_T = \underset{\{\psi | R \in \mathcal{R}\}}{\arg\min}\, [\widehat{\eta}_T - \Psi^{-1}(\psi)]' S_T [\widehat{\eta}_T - \Psi^{-1}(\psi)]' \tag{8}$$

$\widehat{\eta}_T$ from OLS; $S_T = \mathrm{Var}^A(\widehat{\eta}_T)$

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- **Mapping SS to DSGE**: Eqs. (1) and (2) define map $\psi = \mathcal{G}(\theta)$. Estimate $\widetilde{\theta}_T$ via 2-step ALS

$$\widetilde{\theta}_T = \underset{\theta}{\arg\min}\, [\widehat{\psi}_T - \mathcal{G}^{-1}(\theta)]' S_T [\widehat{\psi}_T - \mathcal{G}^{-1}(\theta)]' \tag{9}$$

$\widehat{\psi}_T$ estimated via ML or ALS (with R restrictions), $S_T = \mathrm{Var}^A(\widehat{\psi}_T)$

Inference: *What can we do?*

- D-SVAR enables inference in SVAR and DSGE models. We can test:
  1. Can my DSGE model replicate dynamics of the estimated SS?

$$\begin{cases} \mathbb{H}_0 : \psi = \mathcal{G}(\theta) \\ \mathbb{H}_a : \psi \neq \mathcal{G}(\theta) \end{cases} \implies W_T \sim \chi^2_{\dim(\psi)-\dim(\theta)} \tag{10}$$

Can also test in a subset, e.g. loading matrix F

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     Can also test in a subset, e.g. loading matrix F

  2. How solid is the Identification assumption in my SVAR?

$$\begin{cases} \mathbb{H}_0 : H\text{Vec}(\Pi_{yz}) = 0 \\ \mathbb{H}_a : H\text{Vec}(\Pi_{yz}) \neq 0 \end{cases} \implies W_T \sim \chi^2_r \tag{11}$$

- How reasonable are the assumptions on dynamics of shocks, R?
  - If $\dim(\eta) > \dim(\psi)$, run J-test on (8)

Example: Christiano et al. (1999) - Impact Restrictions SVAR

CEE (1999):

- VAR(4) with real GDP; unemployment rate; CPI; commodity price inflation; and federal funds rate. Sample 1965Q1-2007Q4
- **Identification assumption**: Recursive, in this ordering
- If recursive identification is valid, D-SVAR should recover monetary shock

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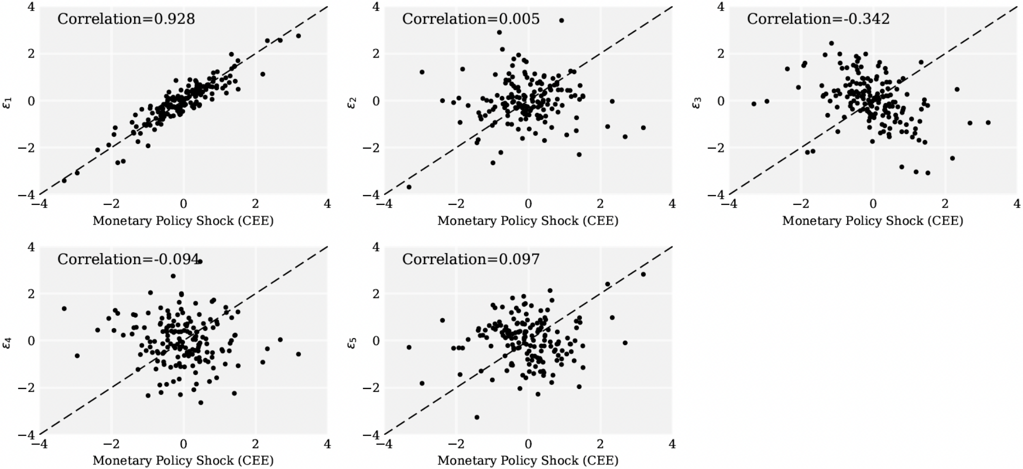
Procedure:

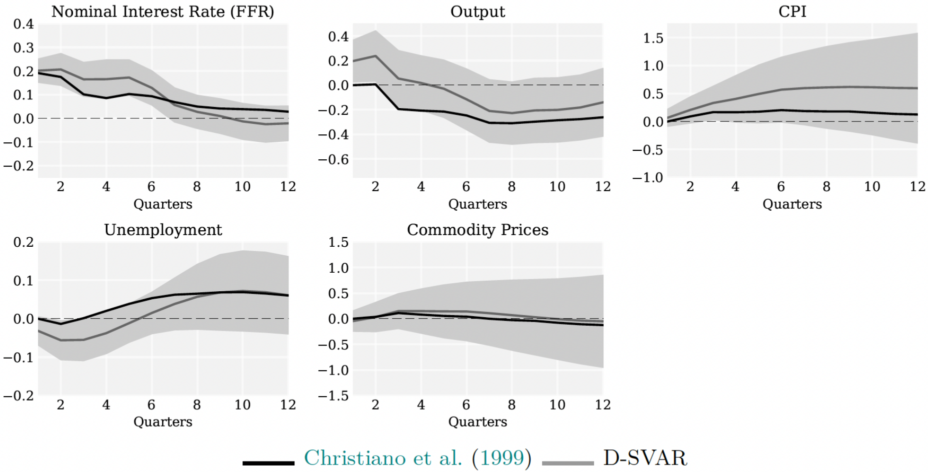
1. Replicate CEE, obtain IRFs of monetary shock + estimated shock ε_t^{CEE}
2. Estimate D-SVAR given restrictions on R, obtain shocks $\widetilde{\varepsilon}_{1,t}, \ldots, \widetilde{\varepsilon}_{5,t}$
3. Check D-SVAR validity: run J-test
4. Compare shocks $\widetilde{\varepsilon}_{1,t}, \ldots, \widetilde{\varepsilon}_{5,t}$ with ε_t^{CEE}: is the monetary shock one of them?

Correlation between CEE (1999) Monetary and D-SVAR shocks



Notes: $\varepsilon_1, \ldots, \varepsilon_5$ are the structural shocks as obtained with the D-SVAR, Sample is 1965Q1-2007Q4

IRFs to CEE (1999) Monetary and D-SVAR shock



Notes: Black lines are IRFs to monetary policy shock as identified by CEE(1999), whereas grey lines IRFs to ε_1 shock. Shaded area represents ± 1 standard deviation around average D-SVAR response obtained from 1000 bootstrap replications.

Testing Recursive Identification

- Does ε_1 satisfy the zero-impact restrictions in CEE (1999)?
- Wald test (11) does not reject recursive restrictions

Table: Test of Zero Impact Restriction (p-values)

	Output	CPI	Unemp.	Com. Price
ε_1	0.2470	0.6587	0.3890	0.8169

Conclusion

Conclusion

- **Key Takeaway**: Standard DSGE assumptions (independent AR(1) shocks) are *sufficient* for identification
  (a) No additional restrictions needed on loading matrix F
  (b) Identification comes from "dynamic fingerprints" in autocovariance structure

- **Practical Implication**:
  (a) Test DSGE cross-equation restrictions
  (b) Evaluate SVAR identification strategies

- **Empirical Examples**: Testing identification strategies of canonical papers in empirical macroeconomics
  (a) CEE (1999): Monetary shock recovered, restrictions validated
  (b) Other examples in paper: Blanchard-Quah (1999); Gertler-Karadi (2015)

Appendix

Mapping: SS $\rightarrow$ Reduced Form (General Version)

- Recast (2) as ABCD form, with state $S_t = \begin{bmatrix} K_t & Z_t \end{bmatrix}'$

$$\begin{aligned} S_{t+1} &= AS_t + B\varepsilon_{t+1} \\ Y_{t+1} &= CS_t + D\varepsilon_{t+1} \end{aligned} \tag{12}$$

- Under invertibility, recover VAR(∞) form:

$$Y_{t+1} = C\sum_{j=0}^{\infty}(A + BD^{-1}C)^j BD^{-1}Y_{t-j} + D\varepsilon_{t+1} \tag{13}$$

- Estimate Reduced-form VAR with innovations a_t:

$$Y_t = \sum_{\ell=0}^{p}\Gamma_\ell Y_{t-1-\ell} + a_t \tag{14}$$

- *Mapping innovations to shocks*: $(\Gamma_1, \ldots, \Gamma_p, \Sigma_a) = \mathcal{H}(\psi)$, where

$$\Sigma_a = DD' \tag{15}$$

Definitions - Spectral Properties

Definition 1 (Auto-covariance Generating Function (ACGF))

Let $\mathbb{E}[Y_t Y_s'] \equiv \Gamma(j;\psi)$ be the autocovariance between $j = s - t$. The ACGF is:

$$\Omega(z,\psi) = \sum_{j=-\infty}^{\infty} \Gamma(j;\psi) z^j$$

set $z = \exp(i\omega)$ an reescale by $(2\pi)^{-1}$ to obtain the spectral density

Definitions - Identification

Let $\mathbf{Y^T} = \{y_t\}_{t=1}^{T}$ be a sample from a distribution $\mathbb{D}(\mathbf{Y^T} \mid \theta)$ with structural parameters $\theta \in \Theta$:

Definition 2 (Global Identification)

$\theta_0 \in \Theta$ is *Globally Identified* if $\forall\, \theta \in \Theta;$ if $\mathbb{D}(\mathbf{Y^T} \mid \theta) = \mathbb{D}(\mathbf{Y^T} \mid \theta_0) \implies \theta = \theta_0$

Definition 3 (Local Identification)

$\theta_0 \in \Theta$ is *Locally Identified* if there exists a neighborhood U_{θ_0} of θ_0 such that $\forall\, \theta \in U_{\theta_0};$ if $\mathbb{D}(\mathbf{Y^T} \mid \theta) = \mathbb{D}(\mathbf{Y^T} \mid \theta_0) \implies \theta = \theta_0$

Usually, Rank conditions are enough to check local identification

Definitions - Identification in State Space

Definition 4 (Observationally Equivalent Processes)

Two state-space representations $\psi, \widetilde{\psi} \in \Psi$ are *observationally equivalent* if they have the same auto-covariance properties, $\Omega(z;\psi) = \Omega(z;\widetilde{\psi})$

Definition 5 (Local Identification in State-Space Representation)

A State-Space system $\mathcal{S}$ is *locally identified* at $\psi \in \Psi$ if there exists U_ψ such that $\forall\, \widetilde{\psi} \in U_\psi$, ψ and $\widetilde{\psi}$ are observational equivalent iff $\psi = \widetilde{\psi}$